High order fluid model for ionization fronts in streamer discharges

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A high order fluid model for streamer dynamics is developed by closing the system after the 4th moment of the Boltzmann equation in local mean energy approximation. This is done by approximating the high order pressure tensor in the heat flux equation through the previous moments. The electric field generated by the space charges is calculated with the Poisson equation. Then planar ionization fronts for negative streamers in N_2 are simulated both with the classical streamer model in local field approximation, and with the present higher order model. The main differences between the models lie in the degree of ionization behind the front and in the mean electron energies.

Streamer discharges occur in nature and as well in many industrial applications such as the treatment of exhaust gasses, polluted water or biogas. They appear when non-ionized or lowly ionized matter is exposed to high electric fields. Here we present a high order fluid model for streamer discharges (that will be derived in detail in a forth coming journal paper), and we use it to simulate planar ionization fronts for negative streamers in nitrogen under normal conditions; and we compare the results with those of the classical fluid model.

The **high order model** is derived by taking the 0th to the 3rd moment of the Boltzmann equation, i.e., by multiplying the Boltzmann equation with the kth power of velocity ($k = 0, 1, 2, 3$) and integrating over velocity space. In principle, the set of moment equations is infinite, but we consider only electron density ($k = 0$), momentum ($k = 1$), energy ($k = 2$) and energy flux ($k = 3$). The system is truncated in the energy flux equation (4) by approximating the high order pressure tensor by the product of lower order moments and by introducing factor of parametrization β. This approximation parameter is chosen as $\beta = 1$ in the present simulations. As a result the hydrodynamical formalization of the streamer dynamics in 1D is described by the nonlinear system of hyperbolic equations

$$\partial_t n + \partial_x(nv) = F_1\,, \quad F_1 = n\nu_I, \tag{1}$$

$$n\partial_t v + \frac{2}{3m}\partial_x(n\varepsilon) - v\partial_x(nv) = F_2\,, \quad F_2 = \frac{nqE}{m} - nv\left[\nu_m + \nu_I\right], \tag{2}$$

$$n\partial_t \varepsilon + \partial_x(n\xi) - \varepsilon\partial_x(nv) = F_3\,, \quad F_3 = nqEv - n\left\{\nu_e\left[\varepsilon - \frac{3}{2}kT_0\right] + \sum_\alpha \nu_{e\alpha}\varepsilon_{e\alpha} + \nu_I\varepsilon + \nu_I\varepsilon_I\right\}, \tag{3}$$

$$\partial_t(n\xi) + \beta\frac{2\varepsilon}{3m}\left(\varepsilon\partial_x n + 2n\partial_x\varepsilon\right) = F_4\,, \quad F_4 = \frac{5qE}{3m}n\varepsilon - \nu_m n\xi. \tag{4}$$

Here n, v, ε and ξ are electron number density, average electron velocity, average electron energy and electron energy flux, correspondingly. E is the electric field and T_0 is room temperature. $\nu_m(\varepsilon)$ and $\nu_e(\varepsilon)$ are the momentum and elastic energy transfer collision frequencies, $\nu_I(\varepsilon)$ is the ionization frequency and $\nu_{e\alpha}(\varepsilon)$ are the collision frequencies for inelastic processes. As charge is conserved, the continuity equation for the ion density n_{ion} is

$$\partial_t n_{ion} = F_1, \tag{5}$$

when the ions are approximated as immobile. Space charge effects are taken into account through the Poisson equation

$$\partial_x E = \frac{e}{\varepsilon_0}(n_{ion} - n)\,, \tag{6}$$

where ε_0 is the dielectric constant and e is the elementary charge.

In essentially all numerical fluid models for streamers in the past 30 years, except for [1, 2], the electron density is approximated by a reaction drift diffusion approximation

$$\partial_t n - \partial_x(\mu E n + D\partial_x n) = F_1\,, \tag{7}$$

that in fact can be derived by closing the fluid equations after the first moment of the Boltzmann equation. This model is called the **minimal model**; it implies a local field approximation of reaction and transport coefficients.

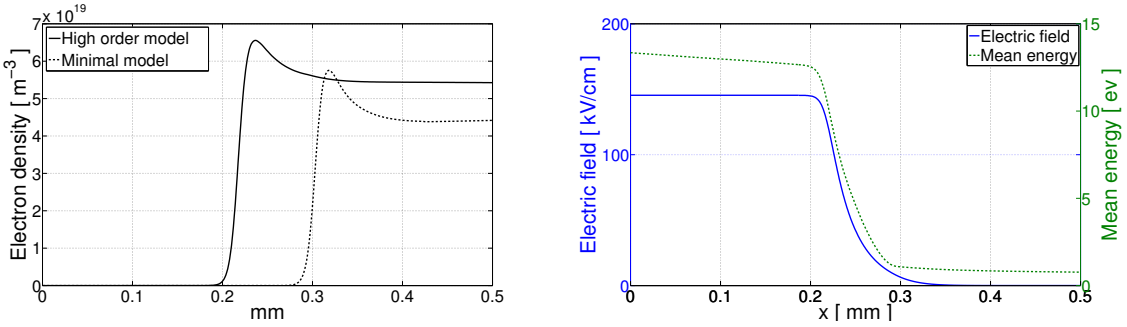

Fig. 1: Left: Electron density profile for the high order model (solid line) and for the minimal model (dashed line), right: mean electron energy (dashed line) and electric field (solid line) profiles in the high order model. The plots show the simulation for instant 0.58 ns for identical initial conditions. The electric field ahead of the ionization front is 145 kV/cm at standard temperature and pressure, which corresponds to 590 Td.

Figure 1 compares the results of the high order model and of the minimal model for the same initial and boundary conditions and for the same electric field ahead of the ionization front. A multi term theory for solving the Boltzmann equation [3] is used to calculate flux transport coefficients and mean-energy dependent collisional rates required as an input in fluid equations.

The following main conclusions can be drawn:

1) While the overall front structure is the same, the electron density behind the front is much larger in the high order model than in the minimal model. That the ionization density was too low in the minimal model, was also concluded from comparisons with Monte Carlo models in [4, 5]. To account for this effect, an extension of the fluid model with a gradient expansion was introduced in [2]. This was a phenomenological approach, while here the new fluid model is derived by a systematic analysis.

2) That the mean electron energy ahead of the front increases while the electric field is constant, was also seen in Monte Carlo simulations before [4], but not yet included in fluid models.

3) The mean electron energy behind the front where the electric field vanishes, is close to 1 eV, because energy relaxation is slow in this region. This feature was not included in fluid models before.

In summary, the new high order fluid model captures effects in streamer simulations that up to now were only inherent in the more microscopic Monte Carlo simulations. This is a step forward for long time calculations.

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